A/AS/IB - Binomial distribution

1. A fair six sided dice is rolled 10 times. Let X be the number of six.
   a. Write down the probability distribution of X
   b. Find the probability that exactly two 6's are rolled
   c. Find the probability that more than two 6's are rolled.

a) $X \sim B(n, p)$
$X \sim B(10, 1/6)$

b) $P(X = 2) = {}^{10}C_2 (1/6)^2 (5/6)^8$ or
using GDC, $0.2907 \approx 0.291$

c) $P(X > 2) = 0.2247 \approx 0.225$

2. A multiple-choice test consists of 25 questions, each with five possible answers. Daniel guesses answers a random
   a. Find the probability that Daniel gets fewer than 10 correct answers
   b. Find the expected number of correct answers
   c. Find the probability that Daniel gets more than the expected number of correct answers.

X = getting a correct answer.
Possibility of 1 correct answer is 1/5

$X \sim B(25, 1/5)$

a) $P(X < 10) = 0.9826 \approx 0.983$

b) $E(X) = np = 25 \times \frac{1}{5} = 5$

c) $P(X > 5) = 0.3833 \approx 0.383$

3. It is known that 1.2% of people in a country suffer from a cold at any time. A company has 80 employees
   a. What assumptions must be made in order to model the number of employees suffering from a cold by a binomial distribution.
   b. Suggest why some of the assumptions might not be satisfied.
      Now assume that the conditions for a binomial distribution are satisfied. Find the probability that on a particular day,
   c. Exactly 3 employees suffer from a cold.
   d. More than three employees suffer from a cold.

Probability of getting a cold = 1·2% = 0·012
$n = 80$

a) All have same probability; employees independent of each other.

b) Cold might be infectious, hence may not be independent.

c) $P(X = 3) = 0.056$

d) $P(X > 3) = 0.015$

4. Eggs are sold in boxes of six. The probability that an egg is broken is 0.06.
   a. Find the probability that a randomly selected box contains at least one broken egg. A customer will return a box to the shop if it contains at least one broken egg. If a customer buys 10 boxes of eggs find the probability that,
   b. She returns at least one box.
   c. She returns more than two boxes.

$n = 6, \quad P = 0.06$

a) $P(X \geq 1) = 0.310$

b) $n = 10, \quad P = 0.310$
$P(X \geq 1) = 0.9755 \approx 0.976$

c) $P(X > 2) = 0.6434 \approx 0.643$

5. An archer has the probability of 0.7 of hitting the target. A round of a competition consists of 10 shots.
   a. Find the probability that the archer hits the target at least seven times in one round.
   b. A competition consists of five rounds. Find the probability that the archer hits the target at least seven times in at least three rounds of the competition.

$n = 10, \quad p = 0.7$

a) $P(X \geq 7) = 0.6496 \simeq 0.650.$

b) $n = 5 \qquad p = 0.6496$

$P(X \geq 3) = 0.7642 \simeq 0.764$

6. Daniel has a biased dice with probability p of rolling a six. He rolls the dice 10 times and records the number of sixes. He repeats this a large number of times and finds that the average number of sixes from 10 rolls is 2.7. Find the probability that in the next set of 10 rules he gets more than four sixes.

Probability of getting a 6 is p.

$n = 10.$

$np = 2.7 \Rightarrow p = \frac{2.7}{10} = 0.27$

$P(X > 4) = 0.1036 \simeq 0.104$

7. A machine produces electronic components that are packaged into packs of 10. The probability that a component is defective is 0.003, independently of all other components.

a. Find the probability that at least one of the components in the pack is defective.
   A pack of 10 is selected at random from the batch. If the pack contains at least one defective component, then another pack is selected from the same batch. If that pack contains at least one defective component then the whole batch is rejected, Otherwise the whole batch is accepted..
b. Find the probability that the batch is rejected.
c. Suggest a reason why the assumptions of independence might not hold.

Shreeji Education
We respect your ability

$n = 10$, $p = 0.003$

a) $P(X \geq 1) = 0.0296$

b) $n = 10$, $p = 0.0296$.

$P(\text{rejected}) = 0.0296 \times 0.0296$
$= 0.00087616$.
$\simeq 0.00087$.

c) defective machine, effects several components