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A machine is designed to generate random digits between 1 and 5 inclusive. Each digit is supposed to appear with the same probability as the others, but Max claims that the digit 5 is appearing less often than it should. To test this claim the manufacturer uses the machine to generate 25 digits and finds that exactly 1 of these digits is a 5.

i Carry out a test of Max's claim at the 2.5% significance level. **[5]**

ii Max carried out a similar hypothesis test by generating 1000 digits between 1 and 5 inclusive. The digit 5 appeared 180 times. Without carrying out the test, state the distribution that Max should use, including the values of any parameters. **[2]**

iii State what is meant by a Type II error in this context. **[1]**

a) $H_0 \sim p = 1/5$ $\quad X \sim B(25, 1/5)$

$H_A \sim p < 1/5$ $\quad P(X=1) = {}^{25}C_1 \, (1/5)^1 \cdot \left(\frac{4}{5}\right)^{24}$

$\alpha = 2.5\%$

$= 0.0236$

$= 2.36\% < 2.5\%$

$\therefore$ claim is not supported

b) $P(X = 180)$ $\quad X \sim B(1000, 1/5)$

approximated to $X \sim N(200, 160)$.

$$\left[\begin{array}{l} np = 1000 \times \frac{1}{5} = 200 \\ npq = 200 \times \frac{4}{5} = 160 \end{array}\right]$$

c). Number 5 dice appear less number to time, and still $H_0$ is accepted.

An engineering test consists of 100 multiple-choice questions. Each question has five suggested answers, only one of which is correct. Ashok knows nothing about engineering, but he claims that his general knowledge enables him to get more questions correct than just by guessing. Ashok actually gets 27 answers correct. Use a suitable approximating distribution to test at the 5% significance level whether his claim is justified. **[5]**

$X \sim B(100, 1/5) \simeq N(20, 16)$ $\quad H_0 \rightarrow \mu = 20$

$\quad H_A \rightarrow \mu > 20$

$\left[\begin{array}{l} np - 100 \times \frac{1}{5} = 20 \\ npq = 20 \times \frac{4}{5} = 16 \end{array}\right] \quad \alpha = 5\%$

$P(X = 27) \Rightarrow z = \frac{26.5 - 20}{\sqrt{16}} = \quad \Rightarrow \varphi = 0.0266$

$[\text{end correction} \rightarrow 27 > 20 \therefore x = 26.5] = 2.6\% < 5\%$

$\therefore$ In sufficient evidence to support claim.

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Jeevan thinks that a six-sided die is biased in favour of six. To test this, Jeevan throws the die 10 times. If the die shows a six on at least 4 throws out of 10, she will conclude that she is correct.

i State appropriate null and alternative hypotheses. **[1]**

ii Calculate the probability of a Type I error. **[3]**

iii Explain what is meant by a Type II error in this situation. **[1]**

iv If the die is actually biased so that the probability of throwing a six is $\frac{1}{2}$, calculate the probability of a Type II error. **[3]**

$X \sim B(10, \frac{1}{6})$  $H_0 \sim p = 1/6$

$P(X \geq 4)$  $H_A \sim p > 1/6$

a) Probability of type I error $\rightarrow P(\alpha)$

$\Rightarrow P(X \geq 4) = 1 - P(X < 4)$

$= 1 - \{P(X=0) + P(X=1) + P(X=2) + P(X=3)\}$

$= 0.0697$

b) The die is biased and not accepting that as less than 4 sixes are thrown so no evidence

c) $X \sim B(10, 1/2)$

$P(X < 4) = 0.172.$

It is claimed that 30% of packets of Froogum contain a free gift. Andre thinks that the actual proportion is less than 30% and he decides to carry out a hypothesis test at the 5% significance level. He buys 20 packets of Froogum and notes the number of free gifts he obtains.

i State null and alternative hypotheses for the test. **[1]**

ii Use a binomial distribution to find the probability of a Type I error. **[5]**

Andre finds that three of the 20 packets contain free gifts.

iii Carry out the test. **[2]**

$H_0 \rightarrow p = 0.30 \quad \alpha = 5\%$

$H_A \rightarrow p < 0.30 \quad n = 20$

$X \sim B(20, 0.30)$

$P(X \leq 2) = 0.0355 < 5\%$

$P(X \leq 3) = 0.1071 > 5\%$

$\therefore$ Probability of type 1 error is 0.0355

$P(X = 3) = 0.0716 > 5\%$

Insufficient evidence to support claim

In the past, the mean time for Jenny's morning run was 28.2 minutes. She does some extra training and she wishes to test whether her mean time has been reduced. After the training Jenny takes a random sample of 40 morning runs. She decides that if the sample mean run time is less than 27 minutes she will conclude that the training has been effective. You may assume that, after the training, Jenny's run time has a standard deviation of 4.0 minutes.

**(a)** State suitable null and alternative hypotheses for Jenny's test. [1]

**(b)** Find the probability that Jenny will make a Type I error. [3]

**(c)** Jenny found that the sample mean run time was 27.2 minutes.

Explain briefly whether it is possible for her to make a Type I error or a Type II error or both. [2]

a) $H_0 \rightarrow \mu = 28.2$ $\quad n = 40$

$H_A \rightarrow \mu < 28.2$

b) $X \sim N(28.2, 4^2)$

$P(X \leq 27)$, $z = \dfrac{27 - 28.2}{4/\sqrt{40}} =$

$\Rightarrow \phi = 0.02869.$

c) $P(X \leq 27.2) > P(x < 27)$

Type II error can be made. and not Type 1 as $H_0$ is rejected in b

A mill owner claims that the mean mass of sacks of flour produced at his mill is 51 kg. A quality control officer suspects that the mean mass is actually less than 51 kg. In order to test the owner's claim she finds the mass, $x$ kg, of each of a random sample of 150 sacks and her results are summarised as follows.

$$n = 150 \qquad \Sigma x = 7480 \qquad \Sigma x^2 = 380\,000$$

**(i)** Carry out the test at the 2.5% significance level. [7]

You may now assume that the population standard deviation of the masses of sacks of flour is 6.856 kg. The quality control officer weighs another random sample of 150 sacks and carries out another test at the 2.5% significance level.

**(ii)** Given that the population mean mass is 49 kg, find the probability of a Type II error. **[5]**

$H_0 \rightarrow \mu = 51$ kg $\quad \alpha = 2.5\%$

$H_A - \mu < 51$ kg

$n = 150$, $\Sigma x = 7480$ $\quad \Sigma x^2 = 380000$

$\bar{x} = \frac{7480}{150} = 49.87$

$\hat{\sigma}^2 = \frac{n}{n-1}\left(\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2\right)$

$= \frac{150}{149}\left(\frac{380000}{150} - (49.87)^2\right)$

$\sigma = 6.828$

$X \sim N(51, 6.828^2)$

$P(X \leq 49.87) =$

$Z = \frac{49.87 - 51}{\frac{6.828}{\sqrt{150}}} = -1.966$

$\phi = 0.0246 = 2.46 \approx 2.5\%$

$\therefore$ sufficient evidence to support claim.

(ii) $\quad X \sim N(49, 6.856)$, $n = 150$

$P(X \geq 49.9) = \frac{49.9 - 49}{\frac{6.856}{\sqrt{150}}} =$

$\phi = 0.0539 \rightarrow$ Probability of type II

The number of cars arriving at a certain road junction on a weekday morning has a Poisson distribution with mean 4.6 per minute. Traffic lights are installed at the junction and a council officer wishes to test at the 2% significance level whether there are now fewer cars arriving. He notes the number of cars arriving during a randomly chosen 2-minute period.

**(a)** State suitable null and alternative hypotheses for the test. [1]

**(b)** Find the critical region for the test. [4]

The officer notes that, during a randomly chosen 2-minute period on a weekday morning, exactly 5 cars arrive at the junction.

**(c)** Carry out the test. [2]

**(d)** State, with a reason, whether it is possible that a Type I error has been made in carrying out the test in part **(c)**. [1]

The number of cars arriving at another junction on a weekday morning also has a Poisson distribution with mean 4.6 per minute.

**(e)** Use a suitable approximating distribution to find the probability that more than 300 cars will arrive at this junction in an hour. [3]

$H_0 \rightarrow \lambda = 9.2$ $\quad \alpha = 2\%$

$H_A \rightarrow \lambda < 9.2$

$X \sim Po(9.2)$ $\quad P(X \le 2) = 0.005$

$P(X \le 3) = 0.0184 < 2\%$

$P(X \le 4) = 0.0486 > 2\%$

∴ Probability of type I error is 0.0184

critical region is $x \le 3$.

$5 > 3$. ∴ $H_0$ is not rejected

No because $H_0$ is not rejected.

$X \sim Po(4.6 \times 60) \approx N(276, 276)$

$P(X \ge 300) \Rightarrow z = \dfrac{300.5 - 276}{\sqrt{276}} =$

[ ]

$\phi = 0.0701$ ---