Linear combination of discrete random variables

Question 1

The random variables X and Y have the independent distributions N(44, 16) and N(30, 9) respectively. Find P (X - Y < 15) .

$X \sim N(44, 16)$ $\quad Y \sim N(30, 9)$.

let $Z' = X - Y$

$Z' \sim N(14, 25)$.

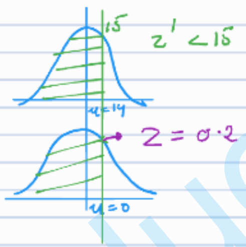
for $P(Z' < 15)$

$Z = \frac{15 - 14}{\sqrt{25}} = 0.2$

$P(Z) = 0.57925$

μ of $Z = 44 - 30 = 14$

$Var(Z) = Var(X) + Var(Y)$
$= 16 + 9 = 25$



Question 2

The random variables W and X have the independent distributions Po(1.2) and Po(2.3) respectively.

(i) Find P(3 ⩽ W +X ⩽ 5)

$W \sim Po(1.2)$ $\quad X \sim Po(2.3)$

let $Z = W + Z$

$Z \sim Po(3.5)$.

$P(3 \le Z \le 5) =$

$\frac{3.5^3 e^{-3.5}}{3!} + \frac{3.5^4 e^{-3.5}}{4!} + \frac{3.5^5 e^{-3.5}}{5!}$

$= e^{-3.5}\left(\frac{3.5^3}{3!} + \frac{3.5^4}{4!} + \frac{3.5^5}{5!}\right)$

$= 0.53676$

(ii) The random variable S is the sum of 100 independent values of W and 200 independent values of X.
Use a suitable approximation to find P(S > 600)

$S = (100W + 200X)$ | $\rightarrow 100 \times 1.2 + 200 \times 2.3 = 580$

$S \sim Po(580)$

$P(S > 600) = 1 - P(S < 600)$

Approximate to Normal with

$S \sim N(580, \sqrt{580})$.

$= 1 - P(S < 600.5)$.

$= 1 - 0.80267$

$= 0.19732$

(iii) The random variable Y has the distribution Po(λ) where $\lambda > 0$. It is given that $\frac{5}{2}P(\lambda = 3) + P(\lambda = 4) = P(\lambda = 5)$.

Find the value of λ.

$\frac{5}{2}(\lambda = 3) + (\lambda = 4) = (\lambda = 5)$

$\Rightarrow \frac{5}{2}\left(\frac{\lambda^3 e^{-\lambda}}{3!}\right) + \frac{\lambda^4 \cdot e^{-\lambda}}{4!} = \frac{\lambda^5 \cdot e^{-\lambda}}{5!}$

$\Rightarrow \cancel{e^{-\lambda}}\left(\frac{5}{2}\frac{\lambda^3}{3!} + \frac{\lambda^4}{4!}\right) = \frac{\lambda^5}{5!}\cancel{e^{-\lambda}}$

$\Rightarrow \frac{5}{2}\frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} - \frac{\lambda^5}{5!} = 0$

$\Rightarrow \frac{\lambda^3}{3!}\left(\frac{5}{2} + \frac{\lambda}{4} - \frac{\lambda^2}{4 \times 5}\right) = 0$

$\Rightarrow \frac{5}{2} + \frac{\lambda}{4} - \frac{\lambda^2}{20} = 0$

$\Rightarrow \frac{50 + 5\lambda - \lambda^2}{20} = 0$

$\Rightarrow -\lambda^2 + 5\lambda + 50 = 0$

$\Rightarrow \lambda = -5, \lambda = 10$ as $\lambda > 0$

$\Rightarrow \lambda = 10$

Question 3

(i) The random variable X has the distribution Po(15). Write down an expression in terms of e for P(X = 12)

$X \sim Po(15)$

$P(X=12) = e^{-15} \frac{15^{12}}{12!}$

(ii) It is given that P (X = n) = P(X = n +1) Write down an equation in n, and hence find the value of n.

$P(X=n) = P(X=n+1)$

$e^{-15} \frac{15^{n}}{n!} = e^{-15} \frac{15^{n+1}}{(n+1)!}$

$\Rightarrow \frac{\cancel{15^{n}}}{\cancel{n!}} = \frac{\cancel{15^{n}} \cdot 15}{\cancel{n!}\,(n+1)}$

$\Rightarrow n+1 = 15$

$\Rightarrow n = 14$

Question 4
The random variables X and Y have independent distributions X ~ Po(3) and Y ~ Po(2) respectively.
(i)Find P(2 < X < 5).

$X \sim Po(3) \quad Y \sim Po(2)$

$P(2 < X < 5) = P(X=3) + P(X=4)$

$= e^{-3}\frac{3^3}{3!} + e^{-3}\cdot\frac{3^4}{4!}$

$= 0.392073$

(ii)Find P(X + Y > 2) .

let $W = X + Y$.

$W \sim Po(5)$

$P(W>2) = 1 - P(W \leq 2)$

$= 1 - \{P(W=0) + P(W=1) + P(W=2)\}$

$= 1 - \{e^{-5} + e^{-5}5 + e^{-5}\frac{5^2}{2!}\}$

$= 0.875$

(iii)The total of 100 random values of X and 150 random values of Y is denoted by T. Use a suitable approximating distribution to find P(T < 560).

$T = 100X + 150Y$

$T \sim Po(100 \times 3 + 150 \times 2)$

$T \sim Po(600)$.

Approximate to Normal.

$T \sim N(600, 600)$.

$P(T < 560)$

1 / 3

$\Rightarrow z = \frac{559.5 - 600}{\sqrt{600}} = -1.65340$

$\phi(z) = 0.0491247$

→ Note:

$P(T \le 560)$ then 560.5

$P(T < 560)$ then 559.5