Note: answers are after the last question.

Question 1
At A factory, A the mean number of accidents per year is 32. At factory B, the records of Numbers of accidents before 2018 have been lost, but number of accidents during 2018 are 21. It is known that the number of accidents per year can be well modelled by Poisson distribution. Use an approximating distribution to test at 2% significance level whether the main number of accidents at factory B is less than at factory A. [6]

Question 2
All the seats on a certain daily flight are always sold. The number of passengers who have bought seats but fail to arrive for this flight on a particular day model by distribution B(320,0.005).
(i) explain what the number 320 represents in this context. [1]
(ii) the total number of passenger who have bought seats but fail to arrive for 2 randomly chose days is denoted by X. Use a suitable-approximating distribution to find $P(2<X<6)$ [3]
(iii) Justify the use of your approximating distribution. [2]

After some changes, the airline wishes to test whether the main number of passengers per day who fail to arrive for this flight has decreased.
(iv) During 5 Randomly chosen days, a total of 2 Passengers failed to arrive. Carry out the test at the 2.5% significance level. [4]

Question 3
On average 1 in 150 components made by certain company are faulty. The random variable X denotes the number of faulty components in a random sample of 500 components.
(i)Describe fully the distribution of X. [1]
(ii)state a suitable approximating distribution. Justify the reason for the same. [2]
(iii)use your approximating distribution to find the probability that sample will include at least three faulty components. [3]

Question 4
Kayoni makes typing mistake while typing a research paper. On an average, she wrongly type 3 words out of 200 words that she types. Let X denotes words that are wrongly typed.
a. Write a suitable distribution to calculate the probability of wrongly typed words in a research paper of 3000 words. [1]
b. Find the probability that she types exactly 52 wrong words while typing the entire research paper. [2]
c. She realises that if she uses a different keyboard, she makes more mistakes while typing and her speed decreases. Use suitable approximation to check the probability of her typing more than 52 wrong words. [3]

Question 5
A random variable X follows a Poisson distribution such that X~Po (37). Show why this distribution can be approximated to Normal distribution.
a. Write the equivalent Normal distribution. [1]
b. Find the probability of X having minimum value of 25 using Normal distribution with end correction. [2]

Question 6
The number of radioactive particles emitted per 150 minute period by some material have a poison distribution with mean 0.7
(i)find The probability that at most 2 particles will be emitted during the randomly chosen 10 hour period. [3]
(ii)Find in minutes the longer time period for which the probability that no particle are emitted is at least 0.99. [5]

Question 7

The number of orders arriving at a shop during an 8 hour working day is modelled by the random variable X with distribution Po(25.2)

(a) State two assumptions that are required for the Poisson model to be valid in this context. [2]

(b) (i)Find the probability that the number of orders that arrive in a randomly chosen 3 hour period is between 3 and 5 inclusive. [3]

(ii)Find the probability that, in 2 randomly chosen 1 hour period, exactly 1 order will arrive in one of the 1 hour periods, and at least 2 orders will arrive in the other 1 hour period. [4]

(c) The shop can only deal with a maximum of 120 orders during any 36 hour period. Use a suitable approximating distribution to find the probability that, in a randomly chosen 36 hour period, there will be too many orders for the shop to deal with. [3]

Let us check

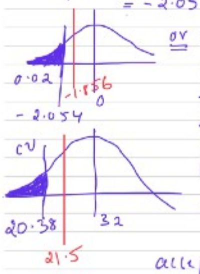
Key

Ans 1

for factory B. $H_0 : \mu = 32$
$H_1 : \mu < 32$.

$B \sim N(32, 32)$.

z for 2% is (using inverse normal) $= -2.054$



or $\Bigg\{$ $z = \dfrac{21.5 - 32}{\sqrt{32}} = -1.856$

$\phi(z) = 0.032 = 3.2\%$

as $3.2 > 2\%$ accept H_0.

as $21.5 > 20.38$ or $-1.856 > -2.054$.

accept H_0.

There is no sufficient evidence to support that accidents have reduced.

Ans 2

(i) The number of passengers who have bought tickets.

(ii) Since $n >> p$ $\lambda = np$
$= 320 \times 0.005$
$= 1.6$ for 1 day.

$\therefore X \sim Po(3.2)$. $\{1.6 \times 2\}$

$P(2 < X < 6) = P(X=3) + P(X=4) + P(X=5)$
$= e^{-3.2}\dfrac{3.2^3}{3!} + e^{-3.2}\dfrac{3.2^4}{4!} + e^{-3.2}\dfrac{3.2^5}{5!}$
$= e^{-3.2}(\quad)$
$= 0.515$.

(iii) $n >> p$ $\lambda = 1.6 < 5$ or $n > 50$

(iv) let passengers who fail to arrive be X.
H_0. $\mu_x = 1.6$
H_1 $\mu_x < 1.6$ $\Big\}$ per day.

for 5 days $\lambda = 1.6 \times 5 = 8.0$

$\therefore X \sim Po(8)$.
$P(X \le 2) = P(X=0) + P(X=1)$.
$= e^{-8}\cdot\dfrac{8^0}{0!} + e^{-8}\dfrac{8^1}{1!} + e^{-8}\dfrac{8^2}{2!}$
$= 0.0138 = 1.38\% < 2.5\%$

Hence reject H_0.

There is evidence to support that passengers failing to arrive has reduced.

Ans 3

(i) $p = 1/150$ $n = 500$

$X \sim B(500, 1/150)$

(ii) $n > 50$, $n >> p$, $np = 500 \times \dfrac{1}{150} = 3.33 < 5$

hence suitable approximation is Poisson. with $X \sim Po(3.33)$

(iii) $P(X \ge 3) = 1 - P(X \le 2)$
$= 1 - \{P(X=0) + P(X=1) + P(X=2)\}$
$= 1 - e^{-3.3}\left\{\dfrac{3.3^0}{0!} + \dfrac{3.3^1}{1!} + \dfrac{3.3^2}{2!}\right\}$
$= 0.6405$
$\simeq 0.641$.

Ans 4

Since λ is high, it can be approximated to Normal with cont correction.

(a) $X \sim N(37, 37)$.

(b) $P(X \geq 24.5) \Rightarrow z = \frac{24.5-37}{\sqrt{37}} = -2.0549$

$\phi(z) = 0.9800.$

Ans 5

(i) $\lambda = 3/200 \Rightarrow$ In 3000 words $\lambda = \frac{3000 \times 3}{200}$

$X \sim Po(45)$

$\left\{ \begin{matrix} 200 & 3 \\ 3000 & ? \end{matrix} \right\}$

(ii) $P(X = 52)$

$= \frac{e^{-45} \cdot 45^{52}}{52!}$

$= 0.03289$

(iii) Approximate to Normal

$X \sim N(45, 45)$.

$P(X \geq 52) = P(X \geq 51.5)$

$Z = \frac{51.5 - 45}{\sqrt{45}} = 0.96896$

$\phi(z) = 0.16629$

Ans 6

$\lambda = 0.7$ per 150 minutes

10 hours $= 10 \times 60 = 600$ minutes.

$\Rightarrow \lambda = 0.7 \times 4 = 2.8$

(i) $\therefore X \sim Po(2.8)$.

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= e^{-2.8}\left(\frac{2.8^0}{0!} + \frac{2.8^1}{1!} + \frac{2.8^2}{2!}\right)$

$= 0.4694$

(ii) $150 \quad 0.7$

$t \quad ?$

λ for t time is $\frac{0.7t}{150}$

$P(X=0) \geq 0.99$

$\frac{e^{-\frac{0.7t}{150}} \left(\frac{0.7t}{150}\right)^0}{0!} \geq 0.99$

$\Rightarrow e^{-\frac{0.7t}{150}} \geq 0.99$

$\Rightarrow -\frac{0.7t}{150} \geq \ln(0.99)$

$\Rightarrow t \leq 2.15$

$\Rightarrow$

Ans 7

a) orders are random
orders are independent
orders are single
orders arrive at constant rate

b) i $8 \rightarrow 25.2$, $3 \rightarrow \lambda$, $\lambda = \frac{25.2 \times 3}{8}$

$\lambda = 9.45$

$X \sim Po(9.45)$ and
$P(3 \le X \le 5)$

$= e^{-9.45}\left(\frac{9.45^3}{3!} + \frac{9.45^4}{4!} + \frac{9.45^5}{5!}\right)$

b(ii) $8 \rightarrow 25.2$, $1 \rightarrow \lambda$, $\lambda = \frac{25.2}{8} = 3.15$

$X \sim Po(3.15)$
$P(X = 1) = e^{-3.15} \frac{3.15^1}{1!}$

$= e^{-3.15}\, 3.15$

$P(X \ge 2) = 1 - P(X < 2)$

$= 1 - \{P(X=0) + P(X=1)\}$

$= 1 - \left\{e^{-3.15} + e^{-3.15} \cdot \frac{3.15^1}{1!}\right\}$

$\therefore P(X=1)$ and $P(X \ge 2)$

$= e^{-3.15} \times 3.15 \times \{1 - e^{-3.15} - e^{-3.15} \times 3.15\}$

$= e^{-3.15} \times 3.15 \times \{- e^{-3.15} - e^{-3.15} \times 3.15\}$

$= 0.11097$

or $P(X \ge 2)$ and $P(X=1)$ {other possibility}

$\therefore$ final ans $= 0.11097 \times 2$
$= 0.220$

c $8 \rightarrow 25.2$, $36 \rightarrow \lambda$.

$\lambda = \frac{36 \times 25.2}{8} = 113.4$

$X \sim Po(113.4)$

$P(X > 120)$

APPROXIMATE TO NORMAL

$X \sim N(113.4, 113.4)$

$z = \frac{120.5 - 113.4}{\sqrt{113.4}} =$

$\phi(z) = 0.252.$